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Small scale intermittency in turbulence

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ABSTRACT. – The present theoretical and experimental knowledge of the intense intermittent events in high Reynolds number turbulence is reviewed. An attempt is made to relate the two main streams of research in this area: the multifractal description and the coherent filaments identified in numerical simulations. It is concluded that, although both approaches can be expressed in a common language, they are inconsistent in detail and both are incomplete. While the multifractal approach is a kinematic description without dynamics, the present understanding of the scaling and dynamics of the coherent vortices requires the existence of stable laminar structures of arbitrarily large Reynolds numbers, and probably represents only one of a hierarchy of intermittent objects. It is suggested that the vortices may not survive at very high values of Re_λ, and that the problem extends to all the intermittent structures of turbulence. It is shown that the filaments should dominate the structure functions in a range of scales that goes from beyond the Kolmogorov scale for $p < 4$ to the whole inertial range if the scaling exponent $\zeta_p \to \infty$ (usually expected as $p \to \infty$.) © Elsevier, Paris

1. Introduction

A central problem in the study of turbulence is to understand the mechanisms by which the rate of energy dissipation becomes asymptotically independent of the magnitude of viscosity, even if dissipation is an intrinsically viscous process. The classical answer is that the energy cascades from larger to smaller eddies until it reaches sizes which are small enough for viscosity to act, at which stage it decays into heat. The cascade becomes longer as the viscosity is made smaller, but a viscous threshold is always reached. The rate of dissipation is controlled by how much energy is fed into the system, and the viscous scale adjusts itself so that all the available energy is eventually discarded.

The quantitative consequences of this process were first formulated by Kolmogorov (1941) who observed that, if the system is to remain in equilibrium, the rate of energy transfer from one eddy size to the next should be the same for all the scales which are larger than the dissipative length η. If we accept on dimensional grounds that the rate of dissipation by an eddy of size ℓ, whose characteristic velocity is u_ℓ, is

$$\epsilon \sim u_\ell^3/\ell, \tag{1}$$

this argument implies that

$$u_\ell \approx (\epsilon\ell)^{1/3}, \tag{2}$$

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leading to the well known $k^{-5/3}$ energy spectrum, and to the estimate of the dissipative scale as $\eta = (\nu^3/\epsilon)^{1/4}$. Equation (2) can also be seen as implying that the velocity field is singular, since the velocity difference between two points at a distance ℓ decays more slowly than for a smooth field, in which $u_\ell \sim \ell$. The equation applies only in the inertial range, $\ell \gg \eta$, and the velocity is smooth and analytic below that threshold, but it becomes singular in the limit of vanishing viscosity, when $\eta \to 0$.

It was realised very soon that the Kolmogorov argument could not be exact, since ϵ is itself a random variable, and (1) and (2) cannot both be averaged at the same time. At any given instant we can expect deviations from either equation, and they cannot be derived directly from the cascade hypothesis. The phenomena connected with this local variability of the dissipation are known by the generic name of *intermittency*.

There are several phenomena grouped under this name, not necessarily related to each other, and almost certainly not universal. Many of them are linked to the large scales of the flow, which are well known to be inhomogeneous in space, so that the cascade is only active where the large scales are present. These large eddies have time and length scales which are much longer than those typical of the inertial range, and they are controlled by instabilities of the mean velocity profiles, which are particular to each type of flow. We will not concern ourselves here with this type of intermittency, usually described by the name of *coherent structures*, even if they are probably the initial stages of the processes which induce intermittency at the smaller scales.

Our interest will be at the other end of the energy cascade, in which the eddies have undergone several cascade stages and have presumably forgotten the initial instabilities. The behaviour of those scales is commonly believed to be universal for all turbulent flows, and the modern formulation of their intermittency characteristics is that the singularity exponent in (2) is itself a random variable, not everywhere equal to 1/3. Consider two neighbourhoods in which

$$u_\ell/u' \sim (\ell/L)^\alpha, \tag{3}$$

and assume that their singularity exponents are different from each other, $\alpha_1 < \alpha_2$. The two quantities u' and L are integral scales associated to the large eddies. The dissipation measured in each neighbourhood would then be

$$\epsilon_\ell \sim u_\ell^3/\ell \sim (\ell/L)^{3\alpha-1} \tag{4}$$

and the ratio of the two dissipations, measured at scale ℓ, would be $\epsilon_1/\epsilon_2 \sim (\ell/L)^{3(\alpha_1-\alpha_2)}$. This ratio grows as the measuring volume is made smaller, and is largest at the smallest distance for which (3) applies. If we assume as a first approximation that this distance is the Kolmogorov scale, $\ell = \eta$, the maximum ratio between the local dissipations is $(\eta/L)^{3(\alpha_1-\alpha_2)}$. If we now define a microscale Reynolds number in terms of the ratio of the largest to the smallest length scales of the flow, $L/\eta = 15^{-3/4}Re_\lambda^{3/2}$, we obtain a scatter for the dissipation which increases with Re_λ as

$$\epsilon_1/\epsilon_2 \sim Re_\lambda^{9(\alpha_2-\alpha_1)/2}. \tag{5}$$

It has been known for several decades (Batchelor and Townsend 1949) that this is true and that, while the probability density function (pdf) of the velocity fluctuations is essentially independent of the Reynolds number, those of the velocity gradients $\nabla u \sim u_\eta/\eta$ are not, and become increasingly intermittent as the Reynolds number is made larger (see figure 1). A recent review of the associated phenomenology has been provided by Sreenivasan and Antonia (1997), and an excellent summary of the theoretical questions involved is given by Frisch (1995).

In this paper we will concern ourselves with a particular problem of the relation between the statistical and structural descriptions of small scale intermittency. In the next two sections we briefly review the statistical aspects, including the quasi-dynamical models that describe the generation of intermittent events in terms of

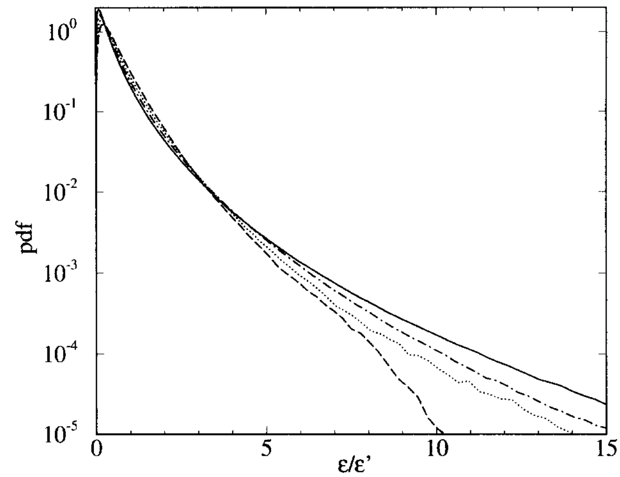

Fig. 1. – Probability density functions of the local dissipation, $|\nabla u|^2$, scaled with its standard deviation. In order of increasing intermittency Re_λ = 36, 64, 96, 168. Simulations by Jiménez *et al.* (1993)

an abstract multiplicative process. In the §4 we summarise the available observations on the structure of small scale vorticity, and their relation with the rest of the velocity gradients. Finally we try to connect both points of view, offer conclusions, and present a list of open problems.

2. The multifractal model

Consider the infinite Reynolds number limit, in which (3) holds down to $\ell = 0$. It is easy to see that the volume V_α filled by the points at which $\alpha < 1/3$ should tend to zero as $\ell \to 0$. Otherwise the total energy dissipated at those points, $\epsilon_\alpha \sim V_\alpha \ell^{3\alpha-1}$, would increase without bound as the measuring scale is made smaller. Mandelbrot (1974) was the first to suggest that the support of the dissipation in turbulent flows might be a self-similar fractal, in the sense that its volume at scale ℓ is given by a power law $V_\ell \sim \ell^{3-D}$. The constant D is the fractal dimension of the set.

There have been many fractal models of turbulence, with varying degrees of physical plausibility and motivation. One of the latest and most general incarnations is the multifractal model, which can be summarised in two hypotheses (Parisi and Frisch 1985)

1 – The local singularity exponent α of the velocity increments varies across the flow, and the points at which it has a given value lie on a self-similar fractal of dimension $D(\alpha)$.

2 – The function $D(\alpha)$ is independent of Reynolds numbers for a given family of flows

A consequence of those hypotheses, and of the further assumption that

3 – the multifractal model holds all the way down to some fixed multiple of the Kolmogorov scale,

is that the characteristics of the pdfs of the velocity gradient as a function of Re_λ can be obtained from the $D(\alpha)$ by letting $\ell/L = \eta/L$.

The multifractal spectrum $D(\alpha)$ is difficult to obtain directly, i.e. by measuring the behaviour of the velocity increments at individual points, but it has been measured indirectly for several turbulent flows by Meneveau and Sreenivasan (1991), who find it to be universal (fig. 2.a). In essence, they measure the way in which the mean value of the different powers of the smoothed velocity gradients change with the size of the smoothing window, and relate it to the multifractal spectrum using a steepest descent approximation. In their experiments they use time traces from single probes, which are approximately equivalent to one dimensional sections of the velocity field. Their dimensions are reduced to three–dimensional space by adding two to all of them, and the values below $D(\alpha) = 2$ are the result of severe extrapolation of sparse experimental data. A single line very seldom intersects anything sparser than a surface.

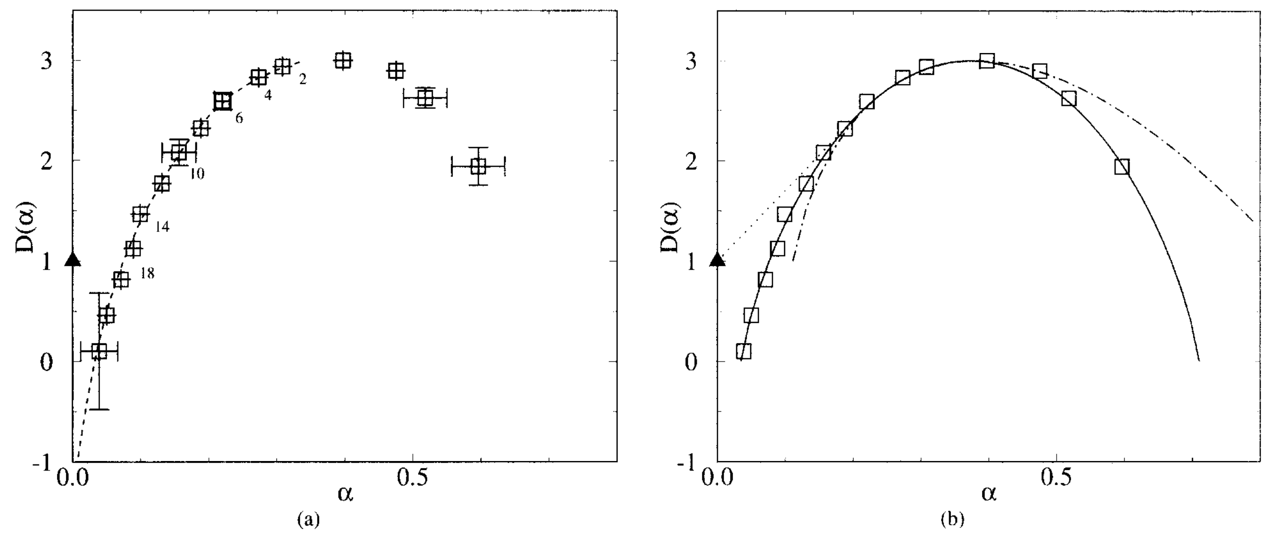

Fig. 2. – Multifractal spectrum for small scales in turbulent flows. The squares and error bars are experimental values from (Meneveau and Sreenivasan 1991). The solid triangle is the result for the coherent vortex filaments (see §5). (a) - - - -: Obtained from experimental multipliers by Chhabra and Sreenivasan (1992). (b) ——: Multiplicative model in (9). — · —: Continuous model by She and Leveque (1994). ········: Convex hull from vortex filaments. Note the minimum dimension predicted by the different models.

The multifractal model has had some success. One common indicator of intermittency is the set of scaling exponents of the velocity structure functions. A second one, which can be derived from it using assumption 3 above, is the Reynolds number dependence of the high order moments of the probability densities of the velocity gradients.

Consider the structure function $S_p = \langle u_\ell^p \rangle$, where $\langle\rangle$ stands for averaging. If ℓ lies in the inertial range we expect S_p to vary like

$$S_p(\ell) \sim \ell^{\zeta_p} \tag{6}$$

In the absence of intermittency, it follows from (2) that $\zeta_p = p/3$, but it has been known for some time that the actual exponents fall below this prediction for $p > 3$. It was also realised early that fractal models of dissipation result in lower exponents for the high order structure functions, because they reduce the volume over which the average is taken. In the multifractal model, the contribution to the structure function of those point with a given singularity exponent α behaves as

$$\langle u_\ell^p \rangle \sim u_\ell^p V_\alpha \sim \ell^{\alpha p + 3 - D(\alpha)}. \tag{7}$$

Of all the possible values of α, the dominant one for small ℓ is the one giving the lowest scaling exponent, so that

$$\zeta_p = \min_\alpha [3 + \alpha p - D(\alpha)]. \tag{8}$$

Different structure functions are dominated by sets with different singularity exponents, and different fractal dimensions. In general the higher orders are due to sparser and more singular sets, and the points of the multifractal spectrum which dominate the different structure functions are marked by their respective orders in figure 2.a.

The comparison with experimental values from two different experiments is given in figure 3, together with the prediction from the Kolmogorov (1941) model in the absence of intermittency. The agreement is remarkable, specially considering that the exponents above $p \approx 10$ are controlled by features whose fractal dimensions are below $D = 2$, and which have therefore large experimental uncertainties when measured from one-dimensional sections. The exponents in figure 3 are those associated with longitudinal velocity increments, in which the separation between the two points of measurement is parallel to the velocity component. Herweijer and van de Water (1995) have measured the scaling exponents of the transverse velocity structure functions, and obtain values which are somewhat lower, corresponding to more intermittent distributions, which would fall in figure 3 slightly below the multifractal prediction.

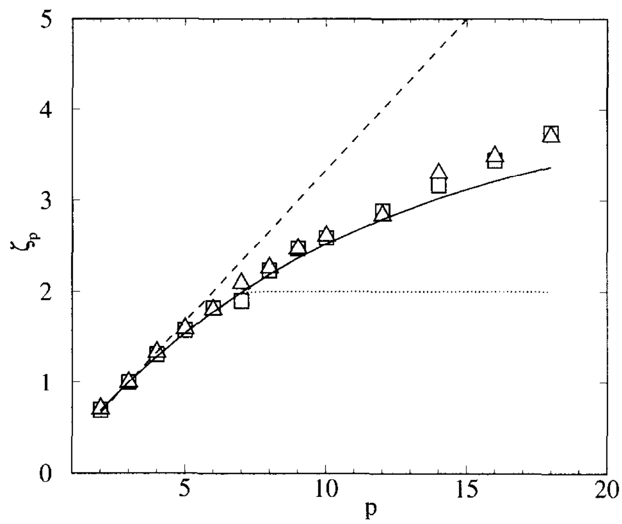

Fig. 3. – Scaling exponents of the longitudinal velocity structure functions. Symbols are experiments. □: Herweijer and van de Water (1994). △: Anselmet *et al.* (1984). $Re_\lambda \approx 500 - 800$ in both cases. - - - -: Kolmogorov (1941). ——: Binomial multiplicative model (9). ········: Convex hull effect from filaments.

The agreement between theory and experiment in figure 3 is impressive, but it is made somewhat less compelling once it is realised that the way in which figure 2.a was obtained is closely related to the direct computation of the structure functions. Analysing the scaling exponents of the powers of the smoothed velocity gradients is not too different from analysing powers of the velocity differences across a given distance. The agreement between the symbols in figures 2.a and 3 is more a consistency check for the theory than a genuine theoretical prediction.

3. Pseudodynamics: multiplicative processes

The first theoretical papers on intermittency (Kolmogorov 1962, Obukhov 1962) already discussed possible generation mechanisms. It was suggested that the energy cascade proceeds stochastically so that, whenever an eddy is broken in two smaller pieces, the energy dissipation is distributed unevenly between them. This led to the prediction that the probability distribution of the dissipation should be lognormal (Gurvich and Yaglom 1967).

Assume that, in the i-th step of the cascade, one part of the eddy receives a fraction $Y_i/2$ of the dissipation. The dissipation per unit volume after n steps would be the product of n random factors, $\epsilon_n = \epsilon_0 \prod_{i=1}^{n} Y_i$, and its logarithm would be the sum of n random variables $\log Y_i$. Assuming that the distribution of those logarithms is independent of the particular step in the cascade, and that it has a finite standard deviation, the central limit theorem implies that the distribution of $\log \epsilon$ would eventually become Gaussian. This argument has been criticised on several counts, specially because the distribution of ϵ in real experiments is not well fitted by a lognormal. It was first noted that the central limit theorem applies only after a large number of steps, which is not the case in real cascades at practical Reynolds numbers, and that it does not apply to the rare events which dominate the high order structure functions, and which constitute the extreme tails of the probability distributions (Novikov 1971, Mandelbrot 1972). In the case of the lognormal distribution, the tails of the logarithm include the limit $\epsilon = 0$, making the application of the theorem even less useful (Kida 1991).

It was also noted that the assumption of finite second moments for the distributions of $\log Y_i$ might not be true, in which case the central limit theorem would predict limits which are not Gaussian. The obvious way in which this could happen would be for Y_i to have a finite probability of being zero, in which case all the moments of the distribution of $\log Y_i$ would be infinite. Several models based on this idea have been proposed (Novikov and Stewart 1964, Frisch, Sulem and Nelkin 1978), mostly as examples of what could go wrong with the lognormal hypothesis. They predict that all the dissipation would eventually be concentrated, with infinite density, on a low-dimensional fractal. Although these models have the advantage of simplicity, they do not fit well the experimental results for the high order structure functions.

More recently Meneveau and Sreenivasan (1987) have observed that the multifractal spectrum obtained in their experiments fits very well the distributions that are derived from a simple binomial multiplicative process, in which each bisection distributes the dissipation into unequal parts, according to fixed coefficients. One half of the eddy receives a fraction $c/2$ and the other one $1 - c/2$. A full cascade step consists of three bisections along the three coordinate planes, and it can be shown that, in the limit of a large number of steps, the multifractal spectrum is given by

$$(9) \qquad \alpha = \frac{1}{3} - s\log_2 c - (1-s)\log_2(2-c), \qquad D(\alpha) = -3[s\log_2 s + (1-s)\log_2(1-s)],$$

where s ranges from 0 to 1, and the parameter c is chosen so as to fit the experimental results. The solid lines in figures 2.b and 3 were obtained from this model with $c = 1.23$.

This simple model can be generalised in several ways. One of the mathematically most attractive is to relax the requirement of discrete scaling steps. When the cascade process is formulated in terms of a continuous semigroup of infinitesimal scale changes, in each of which the dissipation is multiplied by a random factor with a given probability distribution, the results can be related to the theory of infinitely divisible distributions (Novikov 1994). Many models based on this idea have been proposed, which approximate different parts of the data in figure 2.a with varying success. The best known among them (She and Leveque 1994, Chen and Cao 1995) can be reduced to continuous versions of the binomial process (9), in which at each infinitesimal scaling step the local dissipation is multiplied by one of two possible factors with appropriate probabilities. Despite their mathematical simplicity, it can be shown that these processes are poor representations of the multifractal data in figure 2.a, in the sense that either the minimum or the maximum singularity exponents are unbounded. The multifractal spectrum resulting from the She and Leveque (1994) model is given in figure 2.b.

Perhaps the most pointed criticism of the multiplicative processes summarised above is due to Mandelbrot (1974), who noted first that the dissipation is not a particularly physical variable, and that it is not obvious why it should be conserved locally in the cascade. Kraichnan (1974) had also observed that energy dissipation is a quantity associated to the dissipative range of scales, and that it should not be used in arguments about the inertial range. All the above examples are conservative processes in which an eddy receives a given amount of dissipation which has to be distributed among its descendents. This is loosely related to the idea that energy is conserved in the inertial range and has to be transferred without loss to the next cascade stage. While this is globally true, it is not necessarily a local property, since energy can be transferred among neighbouring eddies by pressure forces. Moreover the dissipation depends on the energy density and on a time scale for the decay of the eddies, and while the former is conserved, there are no fixed rules for the latter. What Mandelbrot observed was that, once the requirement of local conservation is removed, there is a whole range of models that can be constructed, leading to distributions which are much more general than the lognormal.

The multiplicative factors have, however, been measured directly in at least two occasions (Van Atta and Yeh 1975, Chhabra and Sreenivasan 1992), and the results support the idea that the dissipation observed at any given time can be described as the result of a self-similar multiplicative cascade whose factors, which are random numbers in the range $(0, 2)$, have a probability distribution which is independent of the cascade step and uncorrelated from one step to another. A multifractal spectrum can be computed from the measured distributions of the factors. It is displayed in figure 2.a as a dashed line and agrees well with the one obtained by other methods. Chhabra and Sreenivasan (1992) have also shown that the spectrum obtained is independent of the scale factor used to define the multipliers, i.e. of whether they are measured by bisecting the original interval as in (9), or by dividing it into a different number of subintervals. This computing procedure is very powerful, and permits the estimation of the frequency of very rare events, including those with negative dimensions which occur only occasionally in time, and which would not even be found as isolated points in most instantaneous realisations.

This observations give some confidence in the description of the process as a multiplicative cascade, since it is known that many difference underlying processes can result in identical multifractal spectra.

4. Structure and dynamics

The main difficulty with all the models surveyed in the last section is their lack of real dynamical mechanisms. Not only is the choice of the dissipation arbitrary, but no process is suggested for its distribution among the descendent eddies at each cascade step. While these "physics independent" scaling arguments are not necessarily wrong, they are incomplete until they can be related to the particular system at hand.

There is no lack of possible multiplicative processes in the energy cascade. The way in which the Navier–Stokes equations create higher velocity gradients and smaller scales in a three-dimensional incompressible fluid has been known for a long time: the vorticity is amplified by being stretched by the velocity gradient tensor. Consider the equation for the vorticity magnitude, $\omega = |\boldsymbol{\omega}|$,

$$\frac{D\omega^2/2}{Dt} = \boldsymbol{\omega} S \boldsymbol{\omega} + \nu \boldsymbol{\omega} \nabla^2 \boldsymbol{\omega}, \tag{10}$$

where S is the symmetric rate of strain tensor $S_{ij} = (\partial_i u_j + \partial_j u_i)/2$. If we neglect the viscous terms and assume that S is random, isotropic and uncorrelated with the vorticity, (10) becomes a multiplicative diffusion process which, under the proper assumptions, leads to a lognormal probability distribution for ω. That distribution is not steady, and the root mean square vorticity grows exponentially with time unless the viscous terms are taken into account. There have been several attempts to do that, but all of them involve an arbitrary model for the effect of viscosity and are difficult to justify. Until that is done, any inviscid multiplicative model for the cascade has to be considered as incomplete, and treated as ad-hoc.

In the last decade it has become possible to observe directly the effect of vorticity amplification in numerically simulated isotropic turbulence, and to characterise the geometry of at least one component of the intermittent events. This was first attempted experimentally by Kuo and Corrsin (1972), who presented suggestive evidence that the high-gradient regions of the flow look like filaments or ribbons. That was later confirmed for the high-vorticity regions in numerical simulations, first at low Reynolds numbers (Siggia 1981, Kerr 1985), and eventually at all the Reynolds numbers accessible to present computers (Hosokawa and Yamamoto 1990, She, Jackson and Orszag 1990, Ruetsch and Maxey 1991, Vincent and Meneguzzi 1991). Vortex filaments were eventually visualised in the laboratory by Douady, Couder and Brachet (1991).

It should be made clear, however, that none of the numerical simulations mentioned above has an inertial range in the classical sense of separate production and dissipation length scales for the turbulent energy, and that none of them satisfy Kolmogorov's 4/5 law (Kolmogorov 1941), which is the only scaling law for which there is a solid theoretical backing (see figure 4). There is experimental evidence that another of Kolmogorov's predictions for the inertial range, the $k^{-5/3}$ spectrum, is only satisfied, even approximately, for $Re_\lambda > 600-700$ (Saddoughi and Veeravalli 1994). None of the conclusions in this section may therefore apply to the inertial range.

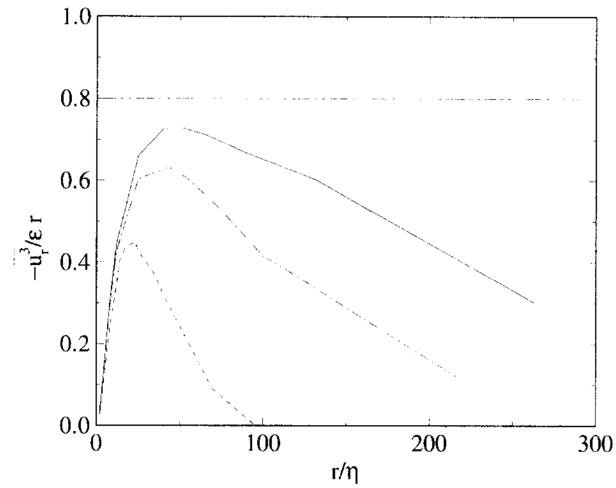

Fig. 4. – Test of Kolmogorov's 4/5 law for the numerical simulations in (Jiménez *et al.* 1993). None of them satisfy it anywhere, and can not be argued to contain an inertial range. ----: $Re_\lambda = 63$, ········: 94, —·—: 143, ——: 168.

The scaling properties of the intermittent filaments were clarified in (Jiménez *et al.* 1993, Jiménez and Wray 1994, Jiménez and Wray 1997) and have been extended since then by other investigators, using both numerical simulations and laboratory experiments. The vortices are long, at least as much as the Taylor microscale and, depending on the definition used for the length, up to the order of the integral scale.

The radial distribution of their axial vorticity is well represented by a Gaussian profile,

$$\omega = \omega_{\max} \exp(-r^2/R^2), \tag{11}$$

which produces a maximum azimuthal velocity $u_\theta \approx 0.3\,\omega_{\max} R$.

A compilation of mean values for R and u_θ is given in figure 5. The implied scalings are

$$R \approx 5\eta, \qquad u_\theta \approx u', \qquad \omega_{\max} \approx 0.3\,\omega_f, \tag{12}$$

where

$$\omega_f = \omega' Re_\lambda^{1/2}. \tag{13}$$

This vorticity is typically much stronger than the rms vorticity ω' of the flow, and the vortices behave as effectively independent from the weaker fluctuations in their surroundings. They are nevertheless approximately in equilibrium between the stretching by the background strain, which is $O(\omega')$, and the viscous diffusion. Indeed in isotropic turbulence the rms longitudinal velocity gradient is $\sigma = (\partial_x u)' = 15^{-1/2}\omega'$ (Batchelor 1953), and the Burgers' radius for a cylindrical vortex in equilibrium with this strain would be $R_b = 2(\nu/\sigma)^{1/2} \approx 4\eta$, in reasonable agreement with (12).

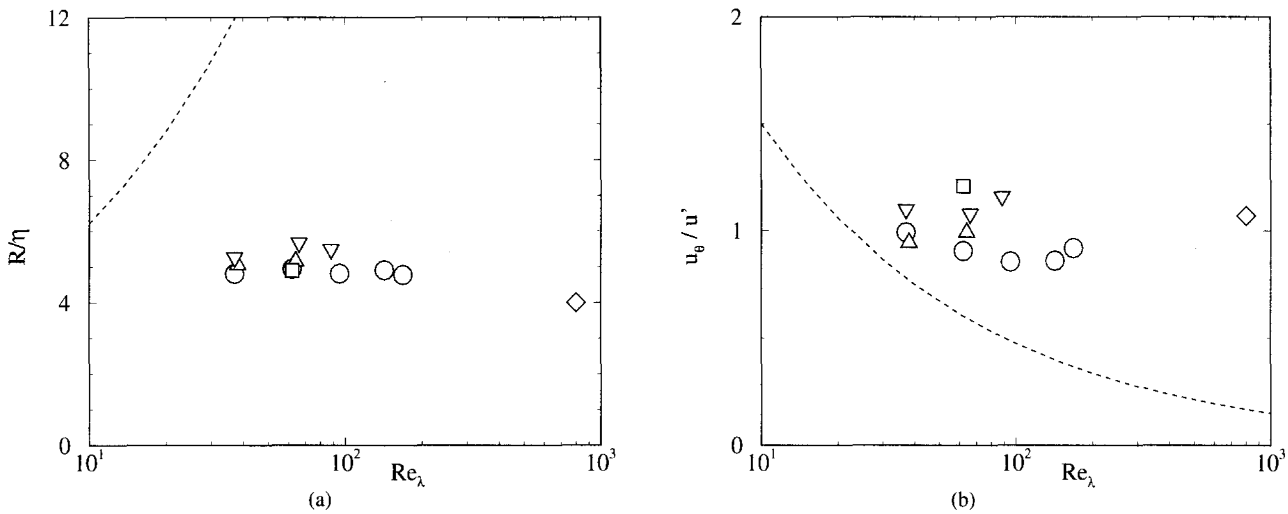

Fig. 5. – Average properties of the vortex filaments. (a) Filament $1/e$ radius, normalised with the Kolmogorov length. The dashed line is the Taylor microscale. (b) Maximum azimuthal velocity, normalised with the large-scale rms velocity. The dashed line is the Kolmogorov inertial velocity increment across $\Delta x \approx 5\eta$. Data from Jiménez and Wray (1997): ○, forced simulations at resolution $k_{\max}\eta = 2$. △, forced simulations at resolution $k_{\max}\eta = 4$. □, decaying simulation at $k_{\max}\eta = 1.4$. ▽: Data from Tanahashi *et al.* (1997a); forced simulations at resolution $k_{\max}\eta = 2.5 - 10$. ◇: Two-component velocity signal from laboratory jet (Malecot and Jiménez, private comm.). ×: One-component velocity signal from counterotating disk experiment by Belin *et al.* (1996) (see Willaime, Belin and Tabeling 1998, for details on data raduction). Some of the scatter among authors is due to differences in the data processing schemes.

Cadot, Douady and Couder (1995) have suggested on theoretical grounds that the vortex radius should scale like the Taylor microscale λ, instead of like η. Their argument is that filaments with lengths of the order of the integral scale L are subject to mean strains of the order of u'/L, whose Burgers' limit is λ, and that the filament cannot be driven to smaller radii. This argument was refuted by Verzicco *et al.* (1995), who showed that even spatially periodic strains with zero average can drive steady vortices of infinite length with radii of the order of the Burgers' limit corresponding to their *rms* strain, rather than to their mean. This conclusion was extended later to unsteady, essentially random, zero-mean strains by Verzicco and Jiménez (1997), and agrees with the estimates in the previous paragraph. In fact, although the axial strain measured along the filaments is $O(\omega')$, it was found by Jiménez and Wray (1994) that it is compressive over a substantial fraction of the length of the filaments, and that it is virtually indistinguishable from the fluctuations of the vorticity stretching term, $\omega S\omega/\omega^2$ over the bulk of the flow.

The scaling (12) of the azimuthal velocity is a priori harder to explain, since stretching a vortex tends to increase its azimuthal velocity, and it could be expected that the maximum velocity excursions would increase with Re_λ as the stretched filaments get thinner. This intermittency effect on the velocities is not observed experimentally (Castaign, Gagne and Hopfinger 1990), and the reason for that was only recently understood to be related to the nonuniformity of the driving strain. Nonuniformly stretched filaments create axial pressure gradients, and axial velocities which tend to resist the nonuniform forcing. An equilibrium is reached when the maximum azimuthal velocity is of the same order as that of the velocity differences in the stretching flow

(Jiménez and Wray 1994, Verzicco *et al.* 1995, Verzicco and Jiménez 1997). The result is that, while stretching results in an amplification of the velocity gradients, there is no equivalent amplification mechanism for the velocities, which are therefore bounded by the maximum available driving velocity difference $O(u')$.

Besides the values compiled in figure 5, Tanahashi *et al.* (1997b) have obtained similar radii and velocities from numerical simulations of a temporally developing mixing layer, and Noullez *et al.* (1997) give examples of $O(u')$ velocity fluctuations over scales of the order of η. The scaling of the largest velocity differences as proportional to u' has been confirmed by Abry *et al.* (1994) and Cadot *et al.* (1995) from data on the pressure fluctuations. On the other hand Cadot *et al.* (1995) present measurements of radii which are much larger than those discussed above ($\approx 10\lambda$), but the discrepancy is large enough to suggest that they are either observing a different phenomenon, or limited by instrumental resolution.

There is convincing evidence that the filaments are formed when stretched vortex sheets, which become much stronger than the vorticity in their neighbourhood, decouple from the background and either roll from their edges or break into individual vortices, in much the same way as in the Kelvin–Helmholtz instability (Vincent and Meneguzzi 1994, Passot *et al.* 1995, Porter, Pouquet and Woodward 1995). This collapse into filaments is enhanced by stretching, in a process that was first observed and studied in free shear layer by Lin and Corcos (1984) and Neu (1984). The filaments resulting from this process cannot be too long, since the coherence length of the strain in a turbulent flow is only of $O(\eta)$, but longer filament can form from the fusion of these short pieces by an axial homogenisation process driven by the same axial pressure waves described above, and which was demonstrated by Verzicco *et al.* (1995).

It can be shown that essentially all the vorticity beyond the threshold defined in (12) is concentrated into coherent filaments, and that the volume fraction filled by filaments which have $\omega_{\max}$ in that range decreases approximately as Re_λ^{-2} (Jiménez and Wray 1997). Indeed if the pdfs for the vorticity magnitude are expressed as

$$p(\omega/\omega') = Re_\lambda^{-5/2} p_0(\omega/\omega_f), \tag{14}$$

the tails of p_0 become independent of Reynolds numbers, at least within the range of the numerical simulations ($Re_\lambda = 36 - 170$). Most of this decrease in volume is due to the filaments getting thinner as $\eta \sim Re_\lambda^{-3/2}$. Since it is known independently that the axes are smooth, and that the ellipticity of the cores tends, if anything, to decrease with increasing Reynolds number (Jiménez *et al.* 1993), this evolution of the volume implies that the total axial length increases as $\mathcal{L} \sim Re_\lambda$.

It is interesting that the same scaling can be used to collapse the pdfs of the longitudinal velocity gradients, suggesting that the objects responsible for both types of gradients are related. Figure 6.a is a compilation of pdfs of longitudinal gradients, spanning several flows and more than two orders of magnitudes in Re_λ. The tails display their characteristic Reynolds number dependence and, despite a considerable amount of noise, are seen to be wider as Re_λ increases. In the right part of the figure, the same pdfs are displayed scaled as in (14). The result is surprising. All the pdfs collapse approximately into a single envelope, highly non-Gaussian and much more intermittent than any of the individual pdfs. This composite pdf cannot be easily described by the stretched exponentials that have proved successful in describing the tails of individual pdfs (Kalislanath *et al.* 1992, Tabeling *et al.* 1996), and is not well approximated either by an algebraic law. Individual pdfs represent only short stretches of the envelope.

Especially disturbing is the short range of the atmospheric boundary layer data in this representation, which only spans $|\partial_x u| = \pm 0.15\, Re_\lambda^{1/2} (\partial_x u)'$. It is easy to show that, for Gaussian vortex cores, the equivalent of (12.c) would be

$$|\partial_x u| \approx 0.2\, Re_\lambda^{1/2} (\partial_x u)'. \tag{15}$$

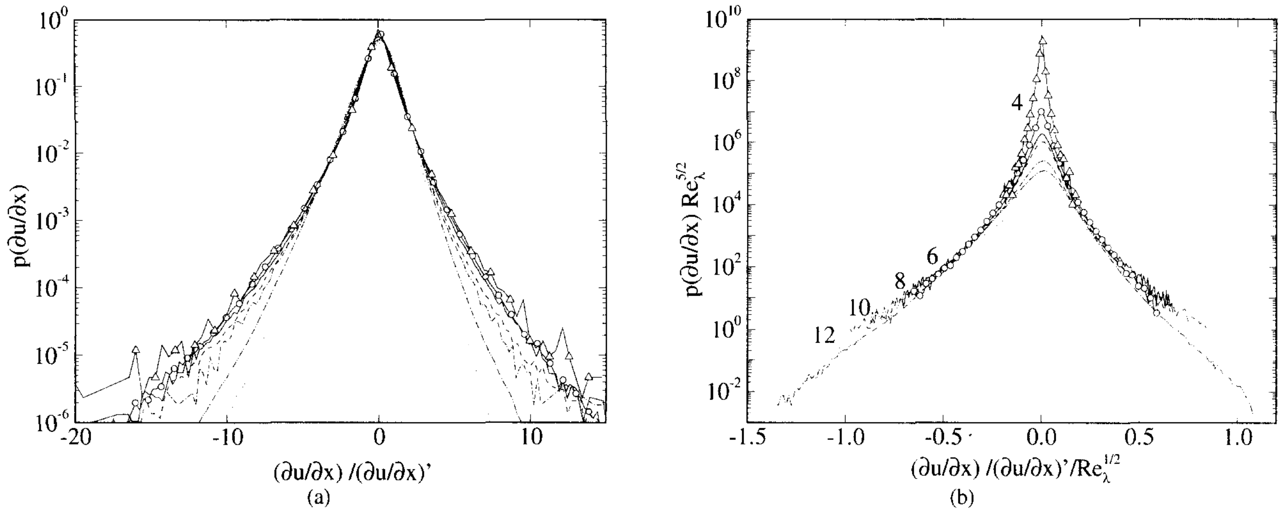

Fig. 6. – Pdfs of the longitudinal velocity gradients for flows at different Reynolds numbers. (a) Normalised with the individual standard deviations. (b) Normalised as in (14). ········ and —··—: $Re_\lambda = 63$ and 143, from the numerical simulations by Jiménez and Wray (1994). —·—, - - - -, —— and —o—: $Re_\lambda = 188$, 328, 407 and 761, from the counterotating disks experiments by Belin *et al.* (1997). —△—: $Re_\lambda = 7200$; atmospheric boundary layer (Antonia, private communication).

This means that atmospheric data may not be sampling the intermittent vortices at all, and that using them for that purpose would require much longer acquisition times (which are not available because of the daily weather cycle†). Figure 6.b also displays the approximate location of the maximum of the integrand in the computation of the different moments of the composite pdf. They are discouraging. For really high Reynolds numbers the compilation of moments beyond $p = 10$ requires samples sizes of the order of $10^{10} - 10^{12}$ *independent* data (see also Meneveau and Sreenivasan 1991). The longest experimental records compiled up to date, in the range of $10^9 - 10^{10}$ samples, are those of Herweijer and van de Water (1995) and Belin *et al.* (1996), and probably represent the limit of reasonable experiments. High order moments are usually studied for the pdfs of velocity differences, rather than of velocity gradients, but the situation is worse there, since the former are less intermittent than the latter. It has been noted by Nelkin (1995) that moments below $p = 10$ are not enough to distinguish among competing models of inertial range intermittency.

From the mean filament vorticity and radius we can obtain a mean vortex circulation, which agrees well with the ones measured for individually tracked filaments (Jiménez and Wray 1994); it increases as $\gamma/\nu \sim Re_\lambda^{1/2}$. Because the normalised circulation of a vortex, γ/ν, is the Reynolds number of its azimuthal velocity field, this increase casts doubt on whether the filaments will remain stable and coherent at very large Re_λ's, at which the azimuthal Reynolds number of individual structures would become correspondingly large. Some indication of such an instability has been observed in the filaments of the direct numerical simulations (Jiménez and Wray 1997), and visual observations of breakdown in the cores of filaments were reported by Douady *et al.* (1991).

The evidence from numerical simulations is indirect, and consist of observations on the axial stretching of the filaments. On the most part, the stretching is $O(\omega')$, consistent with the rms gradients in the background turbulent flow. There is no indication in that observation that the vortices are anything other than approximately straight filaments stretched by the background fluctuations. Any amount of self-interaction would generate a stretching that varies with Re_λ, as the vorticity ω_f of the filaments increases. It turns out that there is a small

† The only planet in our Solar system in which such measurement could be attempted is Venus, which combines a very long day with fast winds and high Reynolds numbers due to its high atmospheric pressure

fraction of the filament length in which the stretching is of the order of ω_f, implying curvatures of the order of the filament radius. Since it can be estimated that the probability of interaction among filaments is negligible, the only way for these large curvatures to appear is for the filaments to become unstable. Straight vortices are stable to weak perturbations, even in the absence of viscosity, but triaxially strained vortices are not, and suffer instabilities with a typical wavelength of the order of their radius (Saffman 1992).

In the range of Re_λ in which the filaments have been studied from simulations, ($Re_\lambda < 170$, $\gamma/\nu < 300$), the fraction of their length subject to instability is small and independent of Re_λ, but there is one experimental, unconfirmed, report that a transition in the intermittency properties of turbulence occurs at $Re_\lambda \approx 700$ (Tabeling *et al.* 1996).

The problem of the Reynolds number of the intermittent structures is not restricted to the filaments. Consider any singular component with exponent α, and compute its Reynolds number when $\ell = \eta$,

$$\frac{u_\eta \eta}{\nu} \sim \frac{u' L}{\nu}\left(\frac{\eta}{L}\right)^{1+\alpha} \sim Re_\lambda^{(1-3\alpha)/2}. \tag{16}$$

For any $\alpha < 1/3$, this Reynolds number increases with Re_λ, and should eventually lead to an instability of the structure. This was first realised by Paladin and Vulpiani (1987), and later interpreted by Frisch and Vergassola (1991) as meaning that η should vary across the flow so that the Reynolds numbers of the individual multifractals would always be $O(1)$. The instability mentioned above might be an instance of this process, but the observations have been made at relatively low Re_λ, and a different behaviour cannot be ruled out at higher Reynolds numbers.

5. Discussion

The power laws satisfied by the filaments suggests that they may represent a particular component of the multifractal hierarchy. The interpretation would be that, as the Reynolds number increases and the viscous cut-off becomes shorter, the strongest filaments represent a set of objects that was previously hidden by the smoothing effect of viscosity.

The $O(u')$ scaling of the azimuthal velocity, independent of the vortex radius, suggests that they represent a flow component in which the velocity difference stays constant as the distance decreases, and which has therefore a singularity exponent $\alpha = 0$. The multifractal point obtained in this way, i.e. a set of filaments ($D = 1$) with $\alpha = 0$, has been included in figure 2. It is inconsistent with the rest of the spectrum. A different argument, which does not depends on using the Reynolds number as a surrogate for the limit $\ell \to 0$, is given by Jiménez and Wray (1997), but the resulting fractal point is the same.

If we take the new point as a *bona fide* component of the multifractal spectrum, equation (8) would imply that $\zeta_p \approx 2$ for all $p > 7$ (figure 3). Equivalently since (8) implies that the multifractal spectrum is convex everywhere, such a point would make all the dimensions below the dotted line in figure 2.b inaccessible to measurement. Since both consequences are incompatible with observations, there is something wrong with the argument.

There is an even more unfavorable interpretation, which is that the vortex radius is a "smoothing" scale applied to an ideal geometrical object formed by the vortex skeletons. The relation between the total vortex length given in the previous section and the Kolmogorov scale, $\mathcal{L} \sim \eta^{-2/3}$, can be interpreted as a fractal dimension, $D = 5/3$. The corresponding fractal point $(0, 5/3)$ is even further away from the rest of the multifractal spectrum, and implies that $\zeta_p = 4/3$ for $p > 4$.

A problem is, as often in turbulence, the commutativity of two limits. The "practical question" is how the properties of turbulence at a given scale ℓ vary as the Reynolds number increases, which is equivalent to the letting $\eta \to 0$. At a given Re_λ the multifractal description corresponds to taking the limit $\ell \to \eta$. The assumption that the order of these two limits is immaterial is not explicit in the multifractal description, but it is usually implied when the formalism is used to address questions about Reynolds number behaviour. It has been used here to deduce scale similarity from the Reynolds number dependence of the gradients. The result suggests that the commutativity is not satisfied, and indeed the question of when to take the inviscid limit is central to the theoretical understanding of turbulence (Constantin 1995).

Consider for example the problem of computing $S_p(\ell)$ for $\ell \gg \eta$. The contribution from the filaments would be $S_p^{(f)} \sim u'^p Re_\lambda^{-2}$, while (6) can be written as $S_p \sim u'^p(\ell Re_\lambda^{-3/2}/\eta)^{\zeta_p}$. The contribution from the filaments dominates if

$$\ell(p) > \eta Re_\lambda^{3/2-2/\zeta_p}. \tag{17}$$

This length is within the inertial range, for large enough Re_λ, if $\zeta_p > 4/3$ or $p > 4$, and tends to the integral scale as $\zeta_p \to \infty$. This implies that there is an intermediate range of length scales between the dissipative and the inertial range in which all the structure functions above S_4 should be independent of ℓ, and that the extent of this range varies for the different moments as in (17), being larger for the higher moments. As far as we know there has been no experimental report of the existence of this range. Note that this prediction is different from the near-dissipation scaling studied by Frisch and Vergassola (1991), which refers to distances below the Kolmogorov scale.

6. Conclusions

We have surveyed the two main approaches to the study of intermittency in isotropic turbulence, both of which are by now fairly well understood. On the one hand we have the multifractal model, which is essentially a kinematic description of the different degrees of intermittency in terms of abstract fractals, whose geometrical structure is not determined. The central result of this approach is the singularity spectrum, which is fitted admirably well by simple multiplicative models for the cascade of turbulent energy. Even if those models have no physical justification, their fit to the experimental spectrum requires an explanation.

On the other hand, the structural study of numerical turbulence fields has given us a clear understanding of a particular component of the intermittent field, in the form of strong vortex filaments.

Strictly speaking both results correspond to different ranges of length scales. While the multifractal model applies to the inertial range, where viscosity can be neglected, the filaments have diameters of the order of the dissipative length, and can be explained in terms of an equilibrium between viscous diffusion and stretching by the background velocity gradients. But the existence of power laws for the scaling properties of the filaments suggest that some version of the multifractal model should apply to them.

The details however are difficult to reconcile, and the most obvious interpretation of the results suggests that the multifractal model has to be modified, at least in the sense that a different scaling must be used in the inertial and the near dissipative ranges. A simple argument leads to the conclusion that the inertial range ends at different scales for different moments, and that there is an intermediate range of scales in which the different structure functions are dominated by the contribution of the coherent filaments. No unified view is however still available, and its formulation remains an increasingly urgent goal for turbulence research.

The problem is not only of theoretical interest. Intermittency has been predominantly studied for isotropic turbulence, but it is a feature of all turbulent flows. The same issues of preferential stretching and rare events are probably behind the lack of wall–unit scaling in the near wall region of turbulent boundary layers (Wei and Willmarth 1989). It has been suggested that the presence of compact vortices is responsible for the increase of flame speed in turbulent combustion (Ashurst, 1994). It is also known that high Rayleigh number convection is subject to intermittent effects in the form of rare strong plumes (Belmonte and Libchaber 1996), which are encountered in the atmosphere as extreme storms, tornadoes and "wind shear". We live, and fly, in a turbulent fluid and, as the population density and the frequency of air travel increase, the rare events which constitute intermittency become a matter of daily concern, and have to be understood.

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